

Fine Tunings for Inflation with Simple Potentials

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Abstract

We continue to investigate possible parameter choices for primordial inflation with simple potentials such as power-law and two-term potentials. We examine the amount of parameter tuning to make the slow-roll inflation eternal. In particular, we adopt the critical coupling for marginally eternal inflation and see whether realistic primordial inflation is attainable under such a parameter choice. It turns out that potentials with mass scale of grand unification provide, after tuning, consistent results to experimental data so that the considered tuning is allowed observationally. Namely, the primordial inflation is possibly marginal in such a setup.

1 Introduction

Landscape of vacua can be regarded as a setup to answer the question how the laws of nature are chosen among possible various theories. In the landscape of vacua, background vacua with nearly flat potentials around them are expected to realize cosmic inflation which results in macroscopic universes out of microscopic quantum fluctuations. From a perspective of a vacuum so chosen, the potential seems to be fine-tuned for flatness of potential. Let us make one-step further assumption that the potential is tuned so that eternal inflation is realized. Is such a physical hypothesis compatible with the observational data?

It depends on representative models we choose whether eternal inflation is consistent or inconsistent to the observational data. For example, the model of the supersymmetric Higgs inflation investigated from this viewpoint in Ref.[1] implies that the spectral index is too small for the case of eternal inflation, while minimal fine tuning is concordant with experimental results. This paper is partially motivated by this negative result on fine tuning for eternal inflation and intended to search a possible realistic example of such a tuning. The example we adopt is simpler than the model of supersymmetric Higgs inflation, which has a particle physics concern of electroweak hierarchy. In particular, we do not assume supersymmetry, though it could be incorporated straightforwardly.

In this paper, we try to mimic GUT potentials with GUT scale mass to consider inflaton potentials with GUT scale ‘mass’ parameter. This ‘mass’ leads to effective intermediate scale after fine tuning of over-all coupling constant. We restrict ourselves to the simplest forms of inflaton potentials such as power-law and two-term potentials postponing investigations on more involved setups for various candidate inflation scenarios [2]. Our present setup allows fine tunings for eternal inflation that is compatible with recent experimental information such as the Planck satellite and BICEP2 results [3, 4]. This gives a concrete example of possible answer to the question how the model parameters are chosen or tuned to realize appropriate slow-roll inflation.

The rest of the paper is organized as follows. In the next section, we present one-parameter tuning for over-all coupling constant with inflaton potentials of power-law form. We recapitulate expressions for inflationary density fluctuations and spectral indices in such a simple case for direct reference. In section 3, we consider parameter tuning to realize eternal inflation within the above setup. In particular, we adopt the critical coupling for marginally eternal inflation [5] and argue that the resultant marginal inflation is compatible with the observational constraints on primordial inflation. In section 4, we comment on the

case of two-term potential through investigation of the polynomial potential similar to GUT potential. The final section concludes the paper.

2 The power-law term

We take the simplest potential of monomial form as a starting point to investigate parameter tuning for eternal inflation.

The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2\lambda^2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2n} v^{4-2n} \phi^{2n}, \quad (1)$$

where the real scalar field ϕ takes values in the range $|\phi| < a$ as the effective theory description is applicable in the present vacuum in the landscape of vacua. The constants v and a are presumably of order one or less in the Planck unit $M_G \simeq 2.4 \times 10^{18} \text{ GeV} = 1$ and the overall coupling λ is the parameter to be tuned for inflation to take place within the regime $|\phi| < a$ around the specified vacuum.

Let us define a rescaled field $\varphi = \phi/\lambda$. Then the kinetic term for the field φ is canonical and its potential is given by

$$V(\varphi) = \frac{1}{2n} \lambda^{2n} v^{4-2n} \varphi^{2n}, \quad (2)$$

with $|\varphi| < a/\lambda$. This monomial form has no sound foundation for its selection so that the present choice is just for simplicity of presentation. From a more general perspective, we have assumed a situation where the single power-law term is dominant when the parameter λ is tuned. The slow-roll inflationary dynamics of the present simple potential is well known [6]. Note that our subject here is not to analyze the dynamics itself but to examine parameter tuning to realize eternal inflation in the present setup. For that purpose, let us recapitulate the known analysis.

Hereafter we restrict ourselves to the regime $\varphi > 0$ without loss of generality. The slow-roll parameters are given by

$$\epsilon(\varphi) \equiv \frac{1}{2} \left(\frac{V'}{V} \right)^2 \simeq \frac{1}{2} \left(\frac{2n}{\varphi} \right)^2, \quad (3)$$

$$\eta(\varphi) \equiv \frac{V''}{V} \simeq \frac{2n(2n-1)}{\varphi^2}. \quad (4)$$

The absolute values of these parameters are smaller than one during inflation.

The inflation ends when the slow-roll parameter ϵ or $|\eta|$ reaches one so that the end point $\varphi = \varphi_f$ of slow-roll inflation is given by

$$\varphi_f \simeq \sqrt{2n}, \quad (5)$$

or

$$\varphi_f \simeq |2n(2n-1)|^{\frac{1}{2}}. \quad (6)$$

The e-fold number \mathcal{N} is given by

$$\mathcal{N} \simeq \int_{\varphi_f}^{\varphi_{\mathcal{N}}} d\varphi \frac{V}{V'}, \quad (7)$$

where $\varphi_{\mathcal{N}}$ denotes the field value of φ corresponding to the e-fold \mathcal{N} . Hence it is obtained as

$$\varphi_{\mathcal{N}} \simeq \sqrt{4n\mathcal{N} + \varphi_f^2} \simeq \sqrt{4n\mathcal{N}}, \quad (8)$$

where we assume \mathcal{N} of several tens and $0 < n \leq 2$.

For direct reference, we here summarize resultant properties of the density fluctuations. The parameters which characterize the primordial inflation model are constrained by observational data. In particular, we have restrictions on the amplitude of density fluctuations $\delta\rho/\rho$ corresponding to the horizon exit of the present horizon with e-fold \mathcal{N} , which is given by

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{5\sqrt{3}\pi} \frac{V^{\frac{3}{2}}(\varphi_{\mathcal{N}})}{|V'(\varphi_{\mathcal{N}})|} \simeq \frac{1}{5\sqrt{3}\pi(2n)^{\frac{3}{2}}} \lambda^n v^{2-n} (4n\mathcal{N})^{\frac{n+1}{2}}. \quad (9)$$

This should turn out to be the observed size of 2×10^{-5} [3]. Then the spectral index n_s is obtained as

$$n_s \simeq 1 - 6\epsilon(\varphi_{\mathcal{N}}) + 2\eta(\varphi_{\mathcal{N}}) \simeq 1 - \frac{n+1}{\mathcal{N}}, \quad (10)$$

and the tensor-to-scalar ratio r is given by

$$r \simeq 16\epsilon(\varphi_{\mathcal{N}}) \simeq 8\frac{n}{\mathcal{N}}, \quad (11)$$

as is well known. The observational data at present seems to suggest $n \lesssim 1$ [3, 4].

3 Inflationary parameter tuning

We now consider possible criteria for inflationary parameter tuning and examine whether such criteria are realistic or not. In particular, we pursue parameter tuning to make slow-roll inflation ‘marginally’ eternal. Namely, we consider marginal inflation [5], where less

tuned potential only realizes non-eternal inflation, or ‘irrelevant’ inflation, and more tuning leads to ‘relevant’ inflation, not to say eternal. The nomenclature is borrowed from that in renormalization theory such as (non-)renormalizable interactions, or (ir)relevant ones.

Realization of slow-roll inflation implies parameter tuning of an inflaton potential in order to make it sufficiently flat. It seems naively that such tuning knows no bounds, since the flatter the potential is, the longer the inflation lasts. How flat is the inflaton potential? Is it almost completely flat?

The minimal requirement of inflationary selection [7] is that the potential is so flat as to induce sizable inflation. Observationally, the potential of the primordial inflation should be flat enough to realize several tens of e -folds.

As a possible further fine tuning, we here consider the parameter tuning to realize eternal inflation. If the fine-tuned parameter λ is exceedingly small, the slow-roll parameters near $\varphi \simeq a/\lambda$ are extremely tiny. In such a case, when φ is very close to a/λ , quantum effects dominate field fluctuations and keep the system in a de Sitter background effectively. This eternal inflation regime exists if quantum variance $\Delta\varphi_q \simeq H/2\pi$ in the field φ during the Hubble time H^{-1} is larger than the corresponding classical change $\Delta\varphi_c \simeq |-V'/3H^2|$, where the Hubble scale $H \simeq \sqrt{V/3}$.

We now present the quantitative condition that a power-law potential supports eternal phase at $\varphi = a/\lambda$. The quantum fluctuations are given by

$$\Delta\varphi_q \simeq \frac{1}{2\pi} \sqrt{\frac{V(a/\lambda)}{3}} \simeq \frac{1}{2\pi\sqrt{6n}} v^{2-n} a^n, \quad (12)$$

On the other hand, the slow-roll during the Hubble time is given by

$$\Delta\varphi_c \simeq \frac{V'(a/\lambda)}{V(a/\lambda)} \simeq \frac{2n\lambda}{a}. \quad (13)$$

The slow-roll tends to be compensated by the quantum fluctuations for $\Delta\varphi_q \gtrsim \Delta\varphi_c$, that is,

$$\frac{1}{2\pi\sqrt{6n}} v^{2-n} a^n \gtrsim \frac{2n\lambda}{a}. \quad (14)$$

The marginal inflation is realized when this condition is marginally satisfied for the ϕ upper bound a of order one to induce inflation. Hence the marginal or critical value of λ to realize eternal inflation in the present setup is obtained through $\Delta\varphi_q \simeq \Delta\varphi_c$ as

$$\lambda_{\text{marg}} \simeq \frac{1}{4\sqrt{6}\pi n^{\frac{3}{2}}} v^{2-n} a^{n+1}. \quad (15)$$

The scales of physical phenomena can be order of magnitude different from one another such as QCD and electroweak scales. It is possible that the largest scale in the effective field theory, where the massive degrees of freedom as ‘heavy’ as the Planck scale have already been ‘integrated out’, is GUT scale. That is, the highest scale (less than the Planck scale) to be dealt with might be GUT scale of order 10^{-2} .

Let us suppose that the scale in the ϕ potential term originates from this largest physical scale. Then the term $\phi^{4-2n}v^{2n}$ gives energy order of magnitude smaller than the Planck scale for $|\phi| < a \lesssim 1$, which is well-described in the framework of effective field theory.

To be specific, we here set the GUT-motivated ‘mass’ scale $v \simeq 1.5 \times 10^{-2}$ with $a \simeq 1$ as sample parameters. Then, by means of Eq.(15), $\lambda_{\text{marg}} \simeq 1.7 \times 10^{-4}, 4.9 \times 10^{-4}, 2.2 \times 10^{-3}$ for $n = 0.5, 1, 1.5$, respectively. These result in the amplitudes of the density fluctuations shown in Eq.(9) as $\delta\rho/\rho \simeq (2.6 - 3.0) \times 10^{-5}, (1.8 - 2.2) \times 10^{-5}, (1.0 - 1.3) \times 10^{-4}$ for $n = 0.5, 1, 1.5$, respectively, under $\mathcal{N} = 50 - 60$. The obtained values happen to fall in the realistic regime (with slight modification of the factor in the scale v , if necessary).

Namely, the considered tuning allows the primordial inflation to be marginally eternal in the present setup. The original GUT scale $v \sim 10^{-2}$ in the term $v^{4-2n}\phi^{2n}$ yields the physical scale in the rescaled term $\lambda^{2n}v^{4-2n}\varphi^{2n}$ of intermediate scale $\lambda^{2n/4-2n}v \sim 10^{-10} - 10^{-3}$ for $\lambda \sim 10^{-3}$ and $n = 0.5 - 1.5$, which is appropriate for the primordial inflation.

Note that the fine tuning $\lambda \sim 10^{-3}$ is chosen not by the GUT scale requirement $v \sim 10^{-2}$ under the observed $\delta\rho/\rho$ but for marginal inflation. The choice $\lambda \sim 10^{-1} - 10^{-2}$ for non-eternal inflation would be sufficient for primordial inflation, whereas $\lambda < 10^{-4}$ would result in eternal inflation with over-tuning.

4 A two-term case

Let us make a remark on the case with two or more terms in the potential. We here adopt the simplest two-term example for presentation. The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2\lambda^2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} v^2 \phi^2 - \frac{1}{4} \phi^4. \quad (16)$$

where the real scalar field ϕ takes values in the range $|\phi| < a$ as the effective theory description is applicable. The constants v and a are again of order one or less in the Planck unit and the overall coupling λ is the parameter to be tuned for inflation within the regime $|\phi| < a$, as is the case for the previous single term example.

We can define a rescaled field $\varphi = \phi/\lambda$ to make the kinetic term for the field φ canonical and then its potential is given by

$$V(\varphi) = \frac{1}{2}\lambda^2 v^2 \varphi^2 + \frac{1}{4}\lambda^4 \varphi^4, \quad (17)$$

with $|\varphi| < a/\lambda$.

The critical coupling for marginal inflation is determined as

$$\lambda_{\text{marg}} \simeq \frac{1}{16\sqrt{3}\pi} a^3, \quad (18)$$

mainly due to the effect of the higher-order ϕ^4 term. If $a \simeq 0.5$, then $\lambda_{\text{marg}} \simeq 10^{-3}$.

The observational value $\delta\rho/\rho \simeq 10^{-5}$ is dominantly realized for $v \simeq 10^{-2}$ by the φ^2 term. More generally, lower-order terms tend to dominate observed density fluctuations, while eternal inflation is mainly controlled by higher-order terms.

The $\lambda^2 v^2 \varphi^2/2$ and $\lambda^4 \varphi^4/4$ terms are comparable in size around $\varphi \simeq 10\sqrt{2}$, which corresponds to $\mathcal{N} \simeq 50$ for φ^2 inflation. This implies that we have a possibility in realistic parameter choice for primordial inflation such that the effects of the φ^4 correction to the φ^2 inflation may be barely seen in the present cosmological observations. For an analysis of inflationary dynamics under such circumstances, see polynomial chaotic inflation investigated in Ref.[8] albeit it deals with supergravity. We conclude that such deviation from the single-term potential may be detectable at the present epoch of our universe if the primordial inflation is marginal.

5 Conclusion

We have investigated possible parameter choices for primordial inflation with simple potentials such as power-law and two-term potentials. We examined the amount of parameter tuning to make the slow-roll inflation eternal. In particular, we adopted the critical coupling for marginally eternal inflation.

It turned out that potentials with mass scale of grand unification provide, after tuning, consistent results to experimental data on cosmological density fluctuations so that the considered tuning is allowed observationally. Moreover, the two-term case implies that the interplay between effects of the two terms may be barely visible around the present horizon scale of our universe. In any case, our setup allows the primordial inflation to be marginally eternal in contrast to the supersymmetric Higgs inflation examined in a previous paper and thus provides a concrete example of marginal primordial inflation.

We note that the power-law and two-term potentials provide just simplest examples which may describe a portion of landscape of vacua. Obviously, more involved setups such as plateau potentials to realize small-field inflation should also be considered. It is expected that future observations discriminate realistic inflation among various possible ones and give us information on parameter choice of nature in primordial inflation. We hope that such knowledge eventually provide a clue to reveal how the laws of nature, in particular, model parameters in particle physics, are chosen among possible various theories.

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